Experimental Substructuring with Application to Joint Identification

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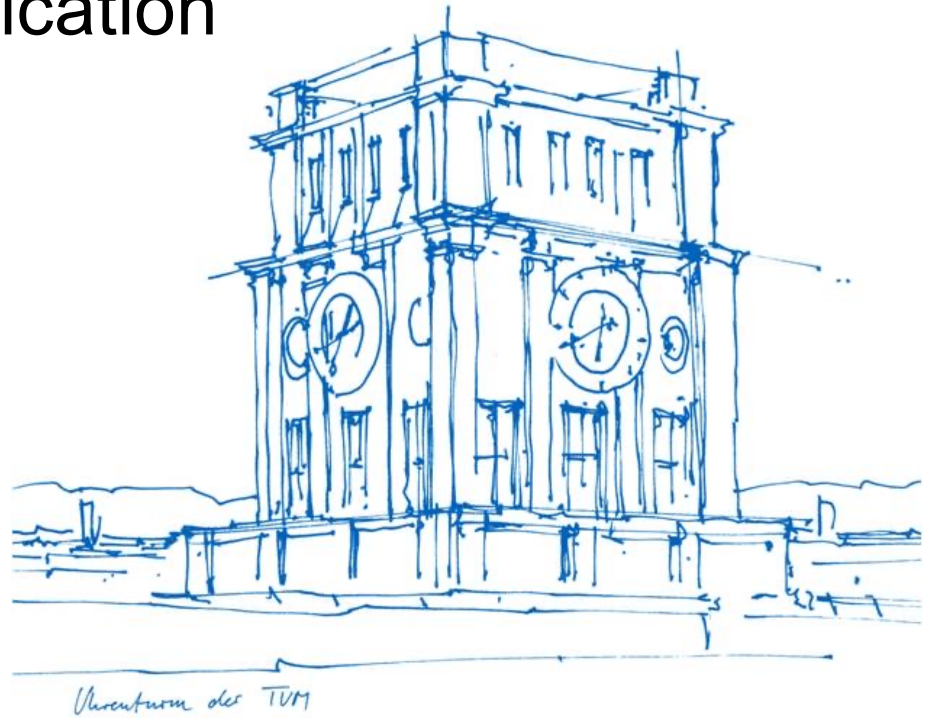


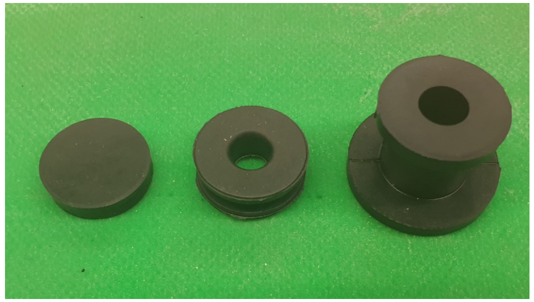
Why not just “measuring the joint”?

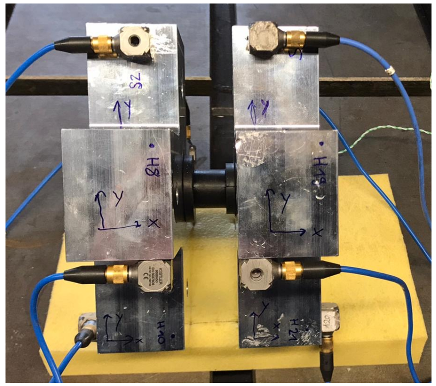
- **Because some “joints” only exist when parts are assembled (e.g. bolted joints)**

- **Because some joints cannot be excited without fixtures (e.g. rubber mounts)**

- **Because in-operation conditions (e.g. preload effect) need to be realistically reproduced to have a representative identification of the joint**

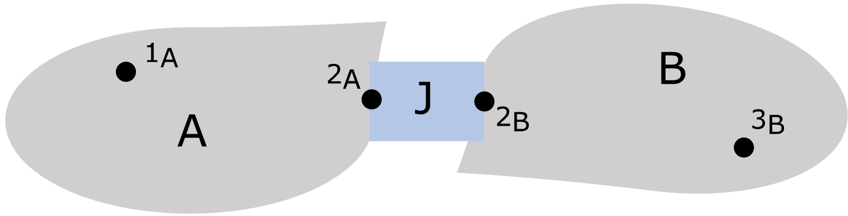
[2]





[2] Brake, Matthew RW, and Pascal Reuß. "The Brake-Reuß beams: a system designed for the measurements and modeling of variability and repeatability of jointed structures with frictional interfaces." *The Mechanics of Jointed Structures: Recent Research and Open Challenges for Developing Predictive Models for Structural Dynamics* (2018): 99-107.

2 steps in joint identification



1. Isolation of the joint

Separating the dynamics of the joints from the dynamics of the assembly

→ Substructuring approach to find $\boldsymbol{Y}^J$ from $\boldsymbol{Y}^{AJB}$

2. Parameterization of joint dynamics

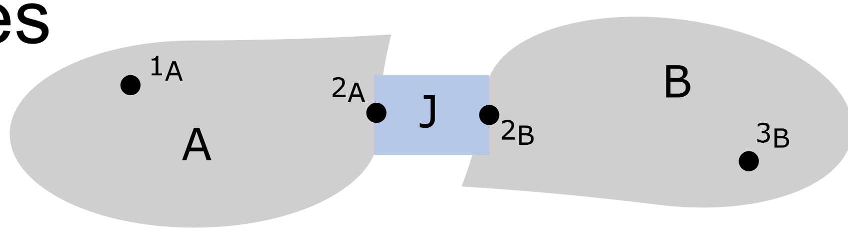
Guess physical parameters from the isolated joint dynamics

$$\boldsymbol{Z}^J(\Omega) = \left(-\Omega^2 \boldsymbol{M}^J + j\Omega \boldsymbol{C}^J + \left(\boldsymbol{K}^J + j\bar{\boldsymbol{C}}^J\right)\right)$$

This presentation is extracted from

On the Robust Experimental Multi-Degree-of-Freedom Identification of Bolted Joints Using Frequency-Based Substructuring, Michael Kreutz, Francesco Trainotti, Verena Gimpl, Daniel J. Rixen (submitted)

Classification of Joint ID techniques based on substructuring



Type of isolation

Type of joint

	Primal	Dual
Dynamic	Primal Decoupling	Dual Decoupling (LM-FBS)
Quasi-static	Inverse Substructuring	LM-FBS with weakened interface

- Optimization and mixing approaches (numerical and experimental like SEMM [1]) not considered here
- We assume 6 Dofs only on each side (VPT)

[1] S. W. B. Klaassen and D. J. Rixen. Using semm to identify the joint dynamics in multiple degrees of freedom without measuring interfaces. In *IMAC-XXXVII: International Modal Analysis Conference, Orlando, FL*, Bethel, CT, January 2019. Society for Experimental Mechanics.

	Primal	Dual
Dynamic	Primal Decoupling	Dual Decoupling (LM-FBS)
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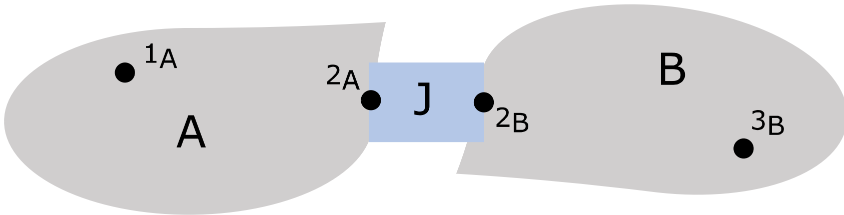

- **Measure admittance of AJB, A and B (then VPT for interface dofs)**

$$\boldsymbol{Y}_{qm}^{AJB}, \boldsymbol{Y}_{qm}^{A}, \boldsymbol{Y}_{qm}^{B}$$

- **Invert to get impedances** $\boldsymbol{Z}_{qm}^{AJB}$, $\boldsymbol{Z}_{qm}^{\dot{A}|B}$

- **“disassemble” A and B**

$$\underbrace{\begin{pmatrix} \boldsymbol{Z}_{2_A,2_A}^{J} & \boldsymbol{Z}_{2_A,2_B}^{J} \\ \boldsymbol{Z}_{2_B,2_A}^{J} & \boldsymbol{Z}_{2_B,2_B}^{J} \end{pmatrix}}_{\boldsymbol{Z}_{qm}^{J}} = \underbrace{\begin{pmatrix} \boldsymbol{Z}_{2_A,2_A}^{A} + \boldsymbol{Z}_{2_A,2_A}^{J} & \boldsymbol{Z}_{2_A,2_B}^{J} \\ \boldsymbol{Z}_{2_B,2_A}^{J} & \boldsymbol{Z}_{2_B,2_B}^{B} + \boldsymbol{Z}_{2_B,2_B}^{J} \end{pmatrix}}_{\boldsymbol{Z}_{qm}^{AJB}} - \underbrace{\begin{pmatrix} \boldsymbol{Z}_{2_A,2_A}^{A} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Z}_{2_B,2_B}^{B} \end{pmatrix}}_{\boldsymbol{Z}_{qm}^{A|B}}$$

	Primal	Dual
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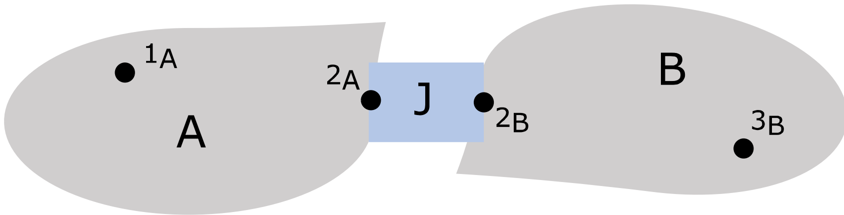

- **Measure admittance of AJB, A and B (then VPT for interface dofs)**

$$\boldsymbol{Y}_{qm}^{AJB}, \boldsymbol{Y}_{qm}^{A}, \boldsymbol{Y}_{qm}^{B}$$

- **Assemble negative A and B substructures to AJB (Decoupling)**

$$\boldsymbol{Y}_{\text{decoupled}}^{J} = \left(\boldsymbol{I} - \boldsymbol{Y}\boldsymbol{B}^{T}\left(\boldsymbol{B}\boldsymbol{Y}\boldsymbol{B}^{T}\right)^{-1}\boldsymbol{B}\right)\boldsymbol{Y} \qquad \text{with} \quad \boldsymbol{Y} = \begin{pmatrix} \boldsymbol{Y}_{qm}^{AJB} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{Y}_{qm}^{A} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{Y}_{qm}^{B} \end{pmatrix}$$

Mathematically equivalent to primal decoupling, unless compatibility and/or equilibrium extended to internal dofs (e.g.[3])

[3] S. Voormeeren and D. Rixen. A family of substructure decoupling techniques based on a dual assembly approach. *Mechanical Systems and Signal Processing*, 27:379–396, 2012.

	Primal	Dual
Dynamic	Primal Decoupling	Dual Decoupling (LM-FBS)
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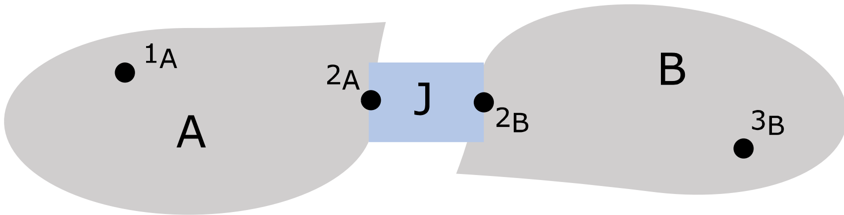


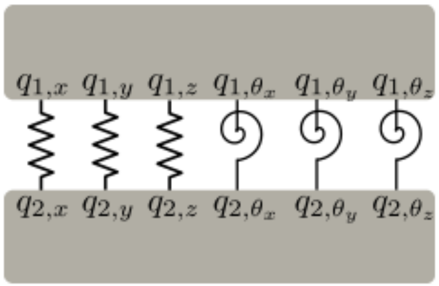
- **Measure ONLY admittance of AJB (then VPT for interface dofs)**

$$\boldsymbol{Y}_{qm}^{AJB}$$

- **Invert to get impedances**

$$\boldsymbol{Z}_{qm}^{AJB} = \begin{pmatrix} \boldsymbol{Z}_{2_A,2_A}^{A} + \boldsymbol{Z}_{2_A,2_A}^{J} & \boldsymbol{Z}_{2_A,2_B}^{J} \\ \boldsymbol{Z}_{2_B,2_A}^{J} & \boldsymbol{Z}_{2_B,2_B}^{B} + \boldsymbol{Z}_{2_B,2_B}^{J} \end{pmatrix}$$

- **and assume dof-to-dof interface topology**



$$\boldsymbol{Z}^{J} = \begin{pmatrix} -\boldsymbol{Z}_{2_A,2_B}^{J} & \boldsymbol{Z}_{2_A,2_B}^{J} \\ \boldsymbol{Z}_{2_B,2_A}^{J} & -\boldsymbol{Z}_{2_B,2_A}^{J} \end{pmatrix}$$

Meggitt, Joshua W. R., et al. "In situ determination of dynamic stiffness for resilient elements." Proceedings of the institution of mechanical engineers, Part C: Journal of mechanical engineering science 230.6 (2016): 986-993.

	Primal	Dual
Dynamic	Primal Decoupling	Dual Decoupling (LM-FBS)
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- **Measure ONLY admittance of AJB (then VPT for interface dofs)**

$$\boldsymbol{Y}_{qm}^{AJB}$$

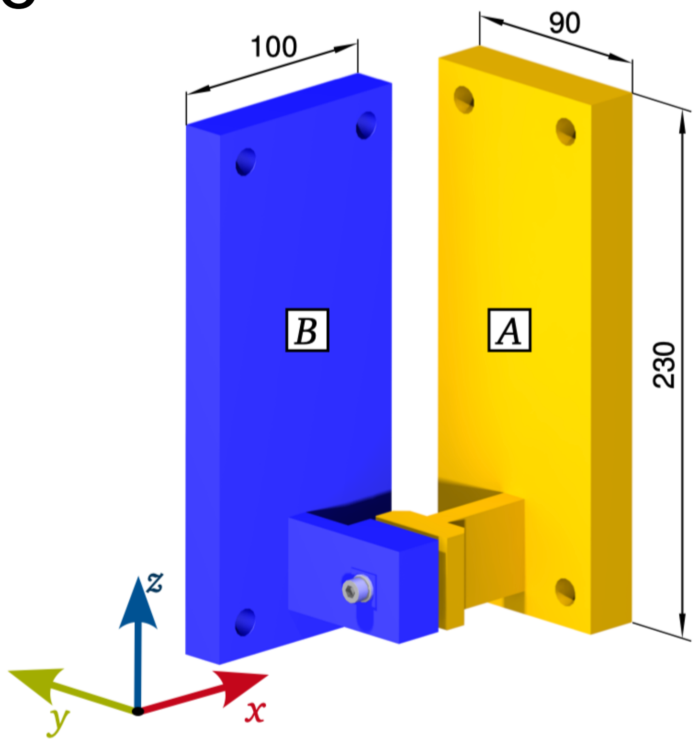
- **From the formula of dual assembly with a "soft" compatibility** $\boldsymbol{Bu} = \Delta\boldsymbol{u}^J = \boldsymbol{Y}^J\boldsymbol{\lambda}$ **isolate** $\boldsymbol{Y}^J$

To much for here …..

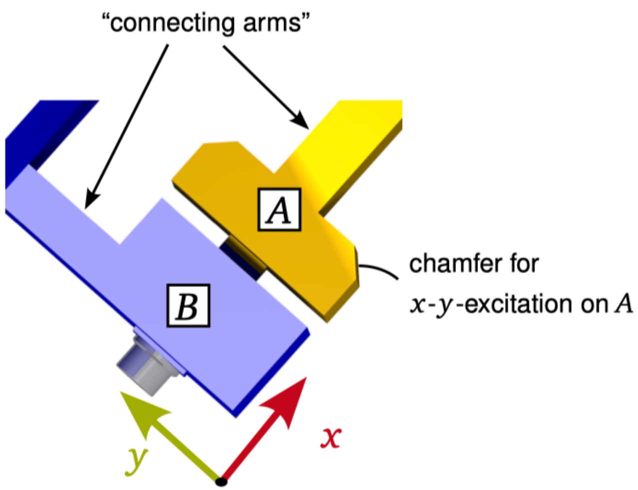
Note: the history of this approach is not clear and there must be some equivalence with the primal inverse substructuring approach, but no proof published so far ….

- *J. Zhen, T. C. Lim, and G. Lu. Determination of system vibratory response characteristics applying a spectral-based inverse sub-structuring approach. part i: analytical formulation. International journal of vehicle noise and vibration, 1(1):1–30, 2004.*
- Čelič, D., & Boltežar, M. (2008). Identification of the dynamic properties of joints using frequency–response functions. *Journal of Sound and Vibration*, *317*(1-2), 158-174.
- *TOL, Şerife, et al. Dynamic characterization of bolted joints using FRF decoupling and optimization. Mechanical Systems and Signal Processing, 2015, 54. Jg., S. 124-138.*

Example



(a) Whole system.



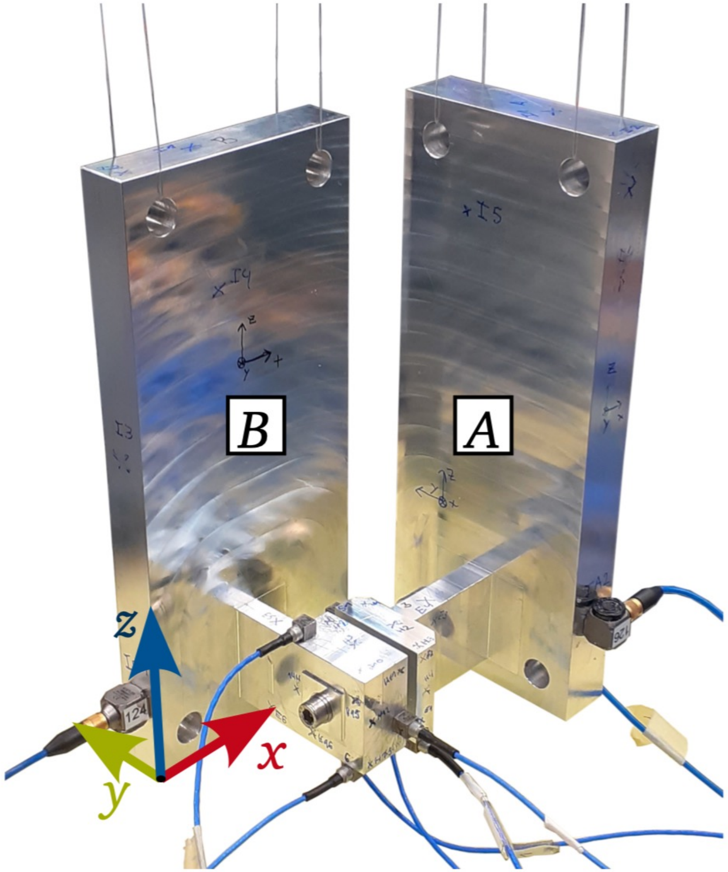
(b) Close-up of the contact.

Data processing
and coupling/decoupling
powered by

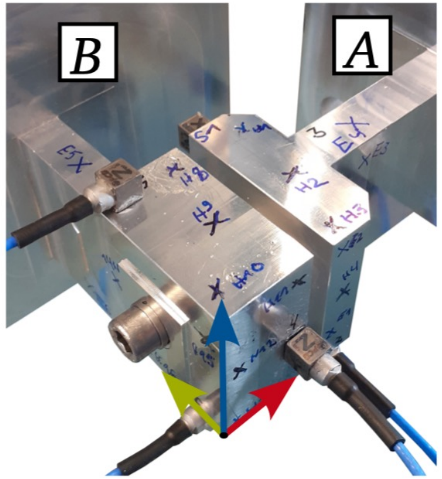
pyFBS

https://gitlab.com/pyFBS/pyFBS

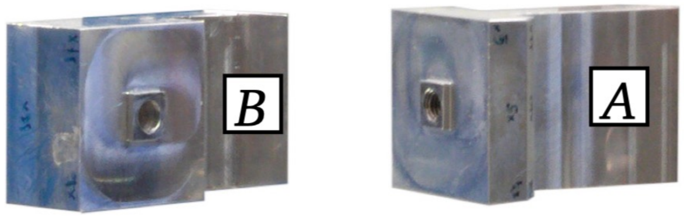
Example



(a) Measurement setup with suspension.



(b) Close-up of the assembled interface.

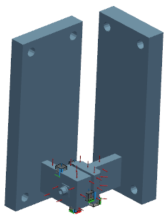


(c) Close-up on disassembled A and B.

Inverse Substructuring

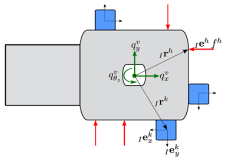
with assemble AJB

Measure assembly AJB



Y_{uf}^{AJB}

Virtual Point Transformation



Y_{qm}^{AJB}

Inversion

$$Z_{qm}^{\mathrm{AJB}} = \left(Y_{qm}^{\mathrm{AJB}}\right)^{-1}$$

Inverse Substructuring

$$\mathbf{Z}_{2,2}^{AJB} = \begin{bmatrix} \mathbf{Z}_{2_A,2_A}^{A} + \mathbf{Z}_{2_A,2_A}^{J} & \mathbf{Z}_{2_A,2_B}^{J} \\ \mathbf{Z}_{2_B,2_A}^{J} & \mathbf{Z}_{2_B,2_B}^{B} + \mathbf{Z}_{2_B,2_B}^{J} \end{bmatrix}$$

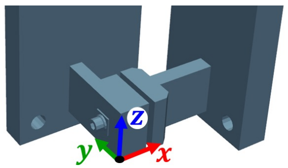
$$\mathbf{Z}_{2_A,2_A}^{J} \approx -\mathbf{Z}_{2_A,2_B}^{J} \approx -\mathbf{Z}_{2_B,2_A}^{J} \approx \mathbf{Z}_{2_B,2_B}^{J}, \qquad \mathbf{Z}_{2,2}^{J} = \begin{bmatrix} -\mathbf{Z}_{2_A,2_B}^{J} & \mathbf{Z}_{2_A,2_B}^{J} \\ \mathbf{Z}_{2_B,2_A}^{J} & -\mathbf{Z}_{2_B,2_A}^{J} \end{bmatrix}$$

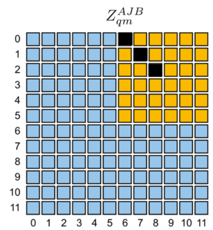
<u>No cross-coupling</u>

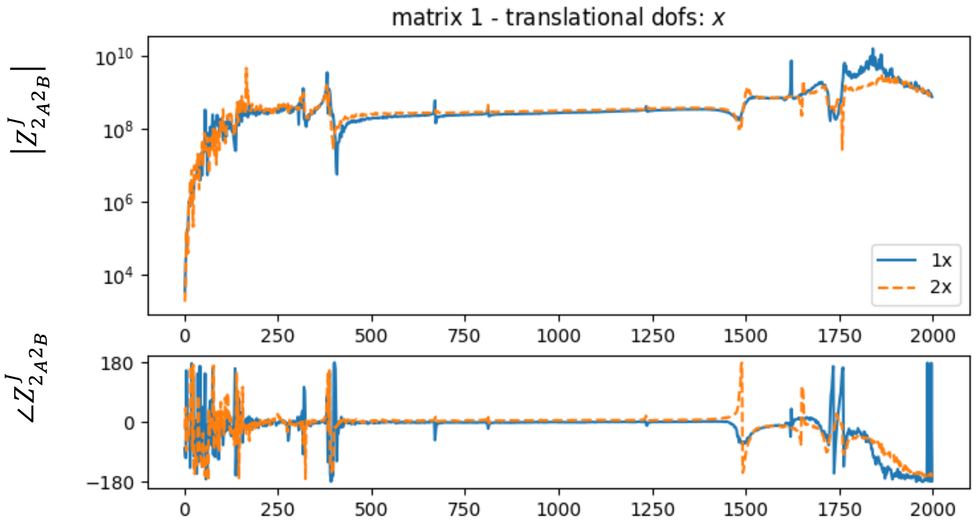
<u>No joint mass</u>

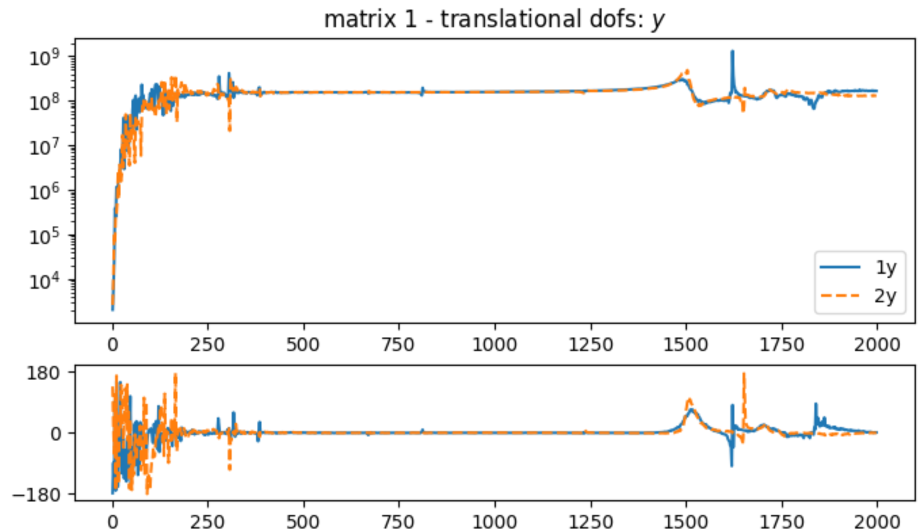
Inverse Substructuring - Results

Interface impedance (including repeatability): translational dofs



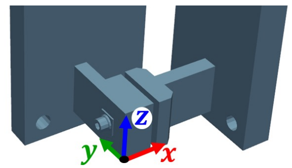


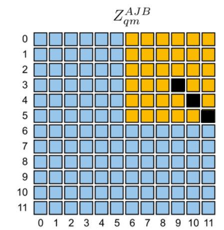


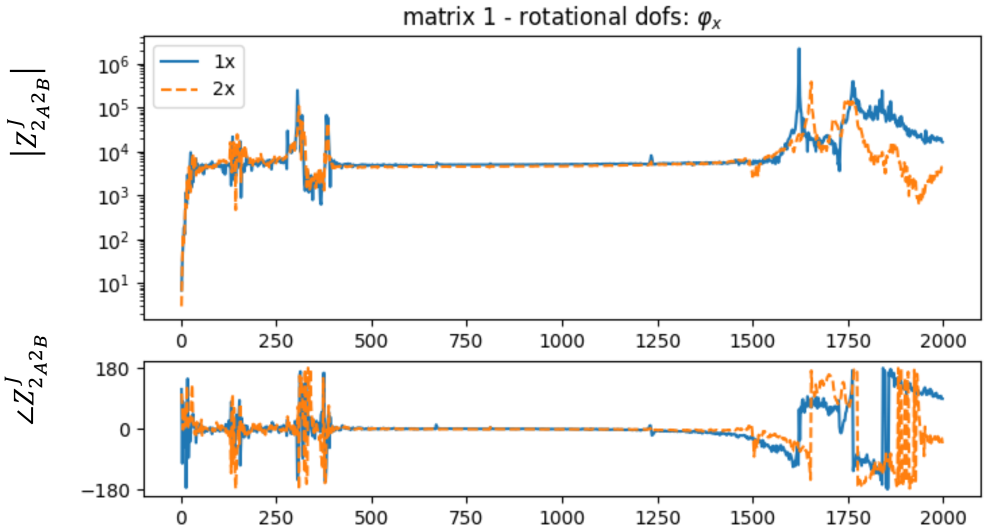


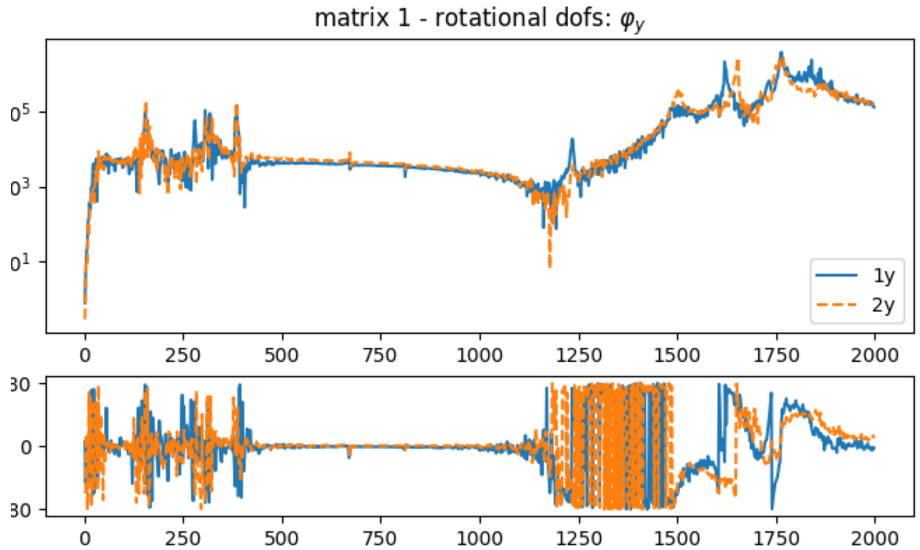
Inverse Substructuring - Results

Interface impedance (including repeatability): rotational dofs





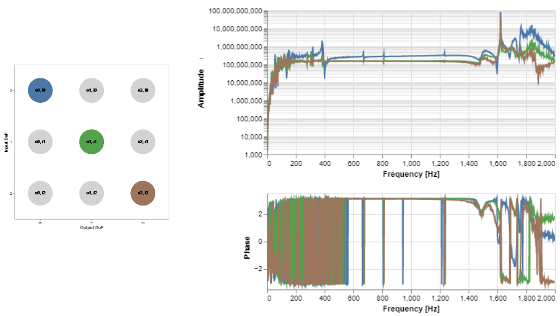




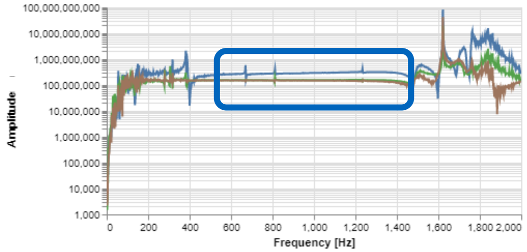
Identification

Result from decoupling
dynamic joint stiffness

$$Z^{\mathrm{J}}_{qm}(\Omega) = \begin{bmatrix} Z^{J}_{2_A 2_A} & Z^{J}_{2_A 2_B} \\ Z^{J}_{2_B 2_A} & Z^{J}_{2_B 2_B} \end{bmatrix}$$



select *clean*
frequency range

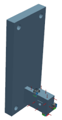


fit parameters
for each direction

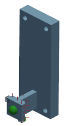
$$Z_{ii}(\Omega) = k_i + j\Omega d_i$$

Validation – Primal Coupling

Measure Subs A, B
+ Virtual Point
Transformation



Y_{qm}^{A}



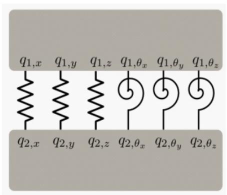
Y_{qm}^{B}

Inversion

$$Z_{qm}^{\mathrm{A}} = \left(Y_{qm}^{\mathrm{A}}\right)^{-1}$$

$$Z_{qm}^{\mathrm{B}} = \left(Y_{qm}^{\mathrm{B}}\right)^{-1}$$

Build $\tilde{Z}^{J}$ from identified parameters
here: **only** stiffness



$$\tilde{Z}_{qm}^{\mathrm{J}}$$

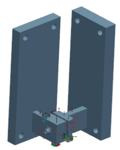
Primal Coupling

$$\tilde{Z}_{qm}^{A\mathrm{JB}} = \tilde{Z}_{qm}^{\mathrm{J}} + Z_{qm}^{\mathrm{A|B}}$$

Inversion

$$\tilde{Y}_{qm}^{A\mathrm{JB}} = \left(\tilde{Z}_{qm}^{A\mathrm{JB}}\right)^{-1}$$

Measure AJB
+ Virtual Point
Transformation



Y_{qm}^{AJB}

Comparison
Validation